Archimedes' Principle

Abel Romer

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1 Organization and Overview

Participants will design rudimentary boats from aluminum foil with the goal of floating as many pennies as possible. In groups of two, participants will be given three 4" squares of aluminum foil and a roll of pennies. The number of pennies that each boat can hold is noted, along with the change in water level. Participants are encouraged to develop theories about the relationship between water level and boat shape, size and weight. A graph is then drawn of all participants' data to show the relationship between the number of pennies and the change in water level. Finally, participants are given 6" and 8" aluminum squares and asked to predict how many pennies a boat made from these might hold. Again, a graph is drawn to show the relationship.

This exploration demonstrates the square-cube law and Archimedes' Principle.

2 Exploration

2.1 Task

In groups of two, participants are given a beaker of water, three 4" squares of aluminum foil, and a roll of pennies. They are challenged to create boats from each of the squares such that each boat can hold as many pennies as possible. They should make a note of the beaker's change in volume and the number of pennies for each boat.

Some questions to consider are:

1. How does the number of pennies affect the change in water level? Is this relationship linear, exponential, or something else?

2. Does the shape of the boat affect the change in water level?

3. What makes a good boat?

2.2 Task

Collect and plot all the data from each of the pairs. What does the relationship look like? Is it what you expected? Can you come up with a rough estimate for a line of best fit?

Can you come up with a general characterization of the relationship between the weight of a floating object and the amount of water it displaces? How does this change if the object sinks?

2.3 Explanation

The mass of displaced water is equal to the mass of the floating object. One way to think about this is to imagine that the water is like a tube of toothpaste and the floating object is like a finger pressing down on the tube. Of course, in this example the masses will be different because all the displaced toothpaste can only exit the tube in one place, but the idea is similar. Once we know the mass m of the displaced water, we can calculate its volume v using

$$v = m/\rho \tag{1}$$

where ρ is the density of water.

This is the basic idea of Archimedes' Principle, which states that the upward, or buoyant, force of the water is equal to the downward force of the floating object. Using our toothpaste analogy, the buoyant force would be the force at which the toothpaste is expelled from the tube.

If the object sinks, then it must be denser than water. In this case, its mass ceases to be important and it simply displaces an amount of water equivalent to its size.

2.4 Task

Pairs of participants are now given 6" and 8" aluminum foil squares and instructed to approximate how many pennies they think a boat made from these can hold. Have the participants create boats from these squares.

Did these new boats hold as many pennies as you expected? More? Less? Can you explain this result? Given what you know about Archimedes' Principle, can you come up with some boat shapes that will hold a lot of pennies?

Now suppose that you have to make submarines, rather than boats. What shape of submarine do you think can hold the most weight without sinking to the ocean floor? Why?

2.5 Explanation

New boats made from larger squares of aluminum will hold significantly more pennies than boats made from smaller squares. This is because the volume of an object increases at a much faster rate than the surface area. For example, imagine a cube with unit side-lengths. If we increase the side lengths by 1, the faces increase in area by 3 square units, and the volume increases by 7 square units.

From what we know about Archimedes' Principle, a good boat would be one that maximizes volume; this is because the greater the volume, the less the average density and the more pennies we can pile on it. What boat-like shapes have the greatest volume?

Here we also want to maximize volume. The difference is that we are now dealing with solids. It's a calculus problem to determine the solid with maximal volume, but it's nevertheless worthwhile to discuss which ones might be better.

3 References

Thanks to Angelica Moreno for the original boat idea, and Ben Ford for the adaptation into a 6th Grade lesson plan.